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BRIEF REPORT

Symbolic Estrangement: Evidence Against a Strong Association Between Numerical Symbols and the Quantities They Represent

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Are numerals estranged from a sense of the actual quantities they represent? We demonstrate that, irrespective of numerical size or distance, direct comparison of the relative quantities represented by symbolic and nonsymbolic formats leads to performance markedly worse than when comparing 2 nonsymbolic quantities (Experiment 1). Experiment 2 shows that this effect cannot be attributed to differences in perceptual processing streams. Experiment 3 shows that there is no additional cost of mixing 2 formats that are both symbolic; that is, the decrement in mixing formats is specific to mixing symbolic and nonsymbolic representations. In sum, we show that accessing a sense of how much a numerical symbol actually represents is a surprisingly difficult and nontrivial process. Our data are consistent with the view that numerical symbols operate primarily as an associative system in which relations between symbols come to overshadow those between symbols and their quantity referents.

Keywords: symbols, number, quantity, representation, comparison

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Does one really have a meaningful sense of very large quantities, like a million or a billion? Or does representing quantities in exact, symbolic form come to change the way one thinks about (and with) these numerical symbols themselves? In recent years, evidence has accumulated in favor of a strong overlap between symbolic and nonsymbolic number representation systems (Dehaene, 1997, 2008; Dehaene, Piazza, Pinel, & Cohen, 2003; Fias, Lammertyn, Reynvoet, Dupont, & Orban, 2003; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; McCrink & Spelke 2010; Nieder & Dehane, 2009; Piazza et al., 2010; Santens, Roggeman, Fias, & Verguts, 2010; Wagner & Johnson, 2011). Considerable attention has been paid to the notion that complex mathematical concepts are grounded in an evolutionarily ancient, fundamental sense of quantity (e.g., which tribe comprises more members, which bush contains more berries; Nieder & Dehaene, 2009; Pica, Lemer, Izard, & Dehaene, 2004). Furthermore, this view proposes that an intuitive sense of approximate quantity (i.e., the approximate number system; ANS) should be a fundamental aspect of any numerical symbol—that is, there should be considerable overlap between symbolic and nonsymbolic numerical processes (Dehaene, 2008). Thus, accessing one's sense of quantity from a symbol should be a relatively fast and effortless process.

On the other hand, it may be that through repeated use and mastery of numerical symbols, the ties between exact numerical symbols (e.g., Arabic numerals) are weakened to the point that these symbols are often used with very little access to a sense of the quantities they presumably represent. For example, it is hard to imagine what a million actually looks or feels like; one's intuitive sense of what 1,000,000 actually means seems divorced from the symbol that is meant to represent that quantity. Of course, we can still use 1,000,000 in myriad ways; for example, it is easy enough to understand that 999,999 < 1,000,000 < 1,000,001. The symbol 1,000,000 is comprehensible in terms of its relative (ordinal) position with respect to other numerical symbols (Verguts & Fias, 2004), even if potentially divorced from the quantity it represents.

A crucial facet of numerical symbols is how they relate to other symbols (Wiese, 2003); indeed, it may even be the case that with repeated exposure to numerical symbols, symbol–symbol relations in literate adults come to usurp symbol–quantity relations. As found in abstract semantic representation more generally (Crutch & Warrington, 2010), how a (numerical) symbol relates to other

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symbols may become more central to that symbol's meaning than how it relates to the quantity it supposedly represents (Deacon, 1997; Nieder, 2009). If so, eliciting a sense of the actual quantity represented by a numerical symbol may be an onerous process, because it is not typically necessary when using such symbols in a normal mathematical context. The link between numerical symbols and the quantities they represent (at least in terms of the ANS) may thus be considerably weaker than previously assumed.

Current Study

One way to distinguish these hypotheses directly is to ask participants to use numerical symbols in a context that forces them to access how much a given symbol represents explicitly. We asked participants to compare quantities represented either in symbolic format (Arabic numeral or written number word) or nonsymbolic format (an array of dots flashed too briefly to count). In Experiments 1–2, participants decided which item depicted the greater quantity in three different conditions: numeral–numeral, dot–dot, and mixed-format (dot–numeral or numeral–dot). In Experiment 3, participants compared quantities in numeral–numeral, number word–number word, and mixed-format (word–numeral or numeral–word) conditions (see Figure 1).

If numerical symbols retain a strong link to an approximate sense of the quantities they represent, then mixing formats should be akin to comparing two entities that ostensibly differ only in representational quality (sharpness of approximate tuning curves; Merten & Nieder, 2009; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). Adults are faster and more accurate when comparing two numeral stimuli than two dot stimuli (Buckley & Gillman, 1974; Lyons & Beilock, 2009). Thus, replacing one dot stimulus with a (superior) numeral stimulus should improve mixed-format comparison performance (relative to dot–dot comparison). According to the hypothesis that symbolic and nonsymbolic quantity representations draw from the same neural populations (Dehaene, 2008; Santens et al., 2010), mixed-format comparisons (which combine a broadly tuned dot stimulus with a finely tuned numeral stimulus) in Experiments 1–2 should yield performance somewhere in between that of numeral–numeral comparisons (two finely tuned stimuli) and dot–dot comparisons (two broadly tuned stimuli). Put more conservatively, mixed performance should at least be no worse than dot–dot comparisons.

By contrast, if symbolic numbers have become detached from an intuitive sense of the nonsymbolic quantities to which they presumably refer, accessing this sense of quantity directly from a numerical symbol may incur an additional processing cost. Hence, mixed-format comparisons should lead to worse performance than either numeral–numeral or dot–dot comparisons. Note that this prediction holds even if numerical symbols and the ANS were never associated to begin with (e.g., Butterworth, 2010; Le Corre & Carey, 2007), though see the Discussion for further consideration of this issue.

In Experiment 3, we tested whether the potential cost of mixing formats observed in Experiments 1–2 might simply be due to mixing representational or visual format, rather than to asymmetric accessing of quantity information. We expected quantities presented as number words to be represented symbolically, as in the case of numerals. We thus predicted that directly comparing a numeral with a number word should not yield performance worse than word–word comparisons (which were expected to yield less efficient performance than numeral–numeral comparisons; Damian, 2004). This result would suggest that the performance degradation seen for mixed comparisons is not simply due to mixing representational or visual formats.

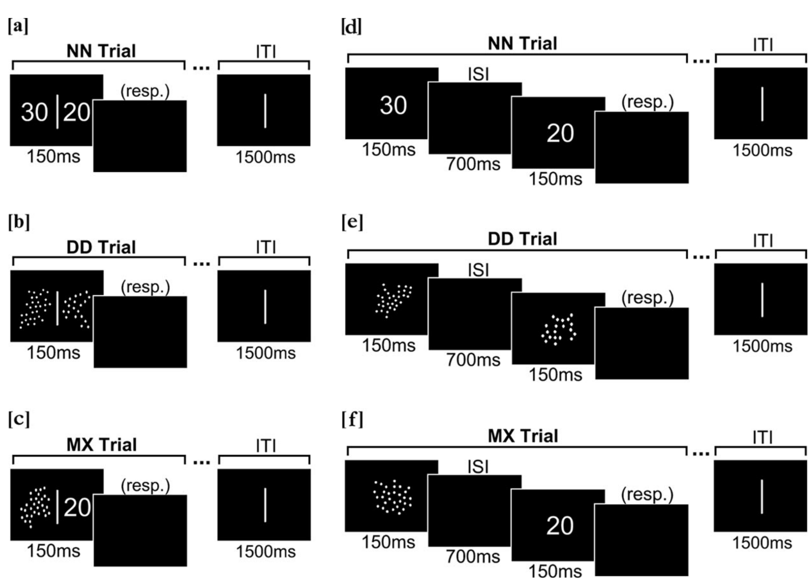

Figure 1. Examples of comparison tasks from Experiment 1 (a–c) and Experiment 2 (d–f). Trial timing was the same for Experiments 2–3. NN = numeral–numeral; DD = dot–dot; MX = mixed-format; ITI = intertrial interval; ISI = interstimulus interval; resp. = response.

Method and Results

Procedures and Stimuli

Participants in Experiment 1 and Experiment 3 were two separate samples of 21 University of Chicago students. Participants in Experiment 2 were 21 Dartmouth College students.

In all experiments, participants' task was to decide which stimulus represented the greater quantity. Participants were to press a key with their left middle finger if they thought the left (Experiment 1) or first (Experiments 2–3) stimulus was greater, press a key with their right middle finger if they thought the right (Experiment 1) or second (Experiments 2–3) stimulus was greater, or press a third key (space bar) with both index fingers if they thought the two stimuli were numerically equal (catch trials). In all experiments, there were 48 critical trials and 16 catch trials (see below) in each condition. Without catch trials, when one had seen 1 or 4 (or 10 or 40) as the first stimulus, the decision would be trivially easy, requiring no further processing of relative quantity. Catch trials were omitted from data analysis (see Goldfarb, Henik, Rubinsten, Bloch-David, & Gertner, 2011, for a discussion of the difficulty in interpreting numerical matching judgments). For critical trials, comparisons were subdivided into four categories: small–far, small–close, large–far, large–close. In all conditions, half of critical trials were numerically *small* (1–4) and half were *large* (10, 20, 30, 40); orthogonally, half of critical trials were numerically *close* ($|n_1–n_2| = 1, 10$) and half were *far* ($|n_1–n_2| = 2, 3, 20, 30$).

For dot–dot trials in Experiments 1–2, half of the arrays in a given comparison pair were equated in terms of overall area (net area of all dots in an array), and the remaining arrays were equated in terms of individual dot area; orthogonally, half of the array pairs were equated in terms of overall contour length (perimeter of the whole array), and the remainder were equated in terms of average local density (distance between neighbors). Pairs equating for each of these parameters were presented randomly (to decrease the likelihood that participants would rely on any one parameter for the duration of the experiment). No array was ever presented to a participant twice. Number words in Experiment 3 were presented in English (centered, 24-point Arial font).

Trials were always blocked by condition (with rest and instructions between blocks and block order randomized across participants). On Experiment 1 mixed-format trials, which side (left or right) contained the dot array was balanced and randomized across trials. In Experiment 2, mixed-format trials were completed in two separate blocks as a function of which stimulus type (dot or numeral) was presented first. Mixed-format performance did not depend on presentation order.

Results

In all experiments, two behavioral measures were collected: response times and error rates. Our hypotheses concerned only the difference between the mixed-format and the single-format condition that yielded the worst performance in each experiment. An examination of means in Table 1 shows that response times and error rates tended to be higher for dot–dot comparisons than for numeral–numeral comparisons in Experiments 1–2 and higher for word–word comparisons than for numeral–numeral comparisons in Experiment 3. Results described below (and shown in Figure 2) therefore focus on the contrast between mixed-format and dot–dot comparisons in Experiments 1–2 and between mixed-format and word–word comparisons in Experiment 3. Note that in all cases in Experiments 1–2 where a significant difference was found between mixed-format and dot–dot conditions, the same was also true for the contrast between mixed-format and numeral–numeral conditions.

Experiment 1. In Experiment 1, response times were significantly longer for mixed-format than dot–dot trials in all categories, $ts(20) \geq 7.97$, $ps < .001$, $ds \geq 1.74$ (see Figure 2, top, white bars). Error rates tended to be higher for mixed-format than dot–dot trials as well: large–close: $t(20) = 1.55$, $p = .137$, $d = 0.34$; large–far: $t(20) = 5.23$, $p < .001$, $d = 1.14$; small–close: $t(20) = 2.30$, $p = .023$, $d = 0.51$; small–far: $t(20) = 2.69$, $p = .014$, $d =$

Table 1
Condition Means in Each Category for Response Times (RT: msec) and Error Rates (ER: % Wrong) for Experiments 1–3 (E.1–E.3, Respectively)

Experiment		Large						Small					
		Close			Far			Close			Far		
		NN	DD	MX	NN	DD	MX	NN	DD	MX	NN	DD	MX
E.1	RT	448	498	924	407	426	812	401	499	799	367	385	754
		27	*28*	*56*	*26*	*23*	*43*	*27*	*30*	*52*	*20*	*15*	*51*
	ER	2.3	22.0	28.5	1.9	1.1	11.0	0.4	6.9	12.4	0.4	0.3	5.9
		1.3	*2.3*	*3.3*	*0.9*	*0.6*	*1.8*	*0.4*	*1.9*	*2.0*	*0.4*	*0.3*	*2.0*
E.2	RT	880	923	1023	873	841	959	842	879	939	813	843	901
		37	*29*	*37*	*40*	*30*	*38*	*37*	*33*	*38*	*41*	*34*	*37*
	ER	5.2	28.6	33.1	6.3	4.4	14.5	5.6	7.9	12.3	2.8	3.2	6.3
		1.8	*2.3*	*3.1*	*1.4*	*1.8*	*2.1*	*1.3*	*2.5*	*2.6*	*1.0*	*1.2*	*1.7*
		NN	WW	MX	NN	WW	MX	NN	WW	MX	NN	WW	MX
E.3	RT	648	754	748	666	728	718	650	704	725	653	699	708
		34	*30*	*28*	*41*	*36*	*30*	*38*	*30*	*31*	*38*	*41*	*26*
	ER	2.8	6.4	5.2	3.2	4.0	4.0	5.6	4.8	5.1	3.2	2.8	2.9
		1.7	*2.8*	*1.3*	*1.2*	*1.6*	*1.9*	*1.8*	*1.7*	*1.4*	*1.1*	*1.1*	*1.5*

Note. Values in italics are standard errors of the mean. NN = numeral–numeral; DD = dot–dot; MX: mixed-format; WW = word–word.

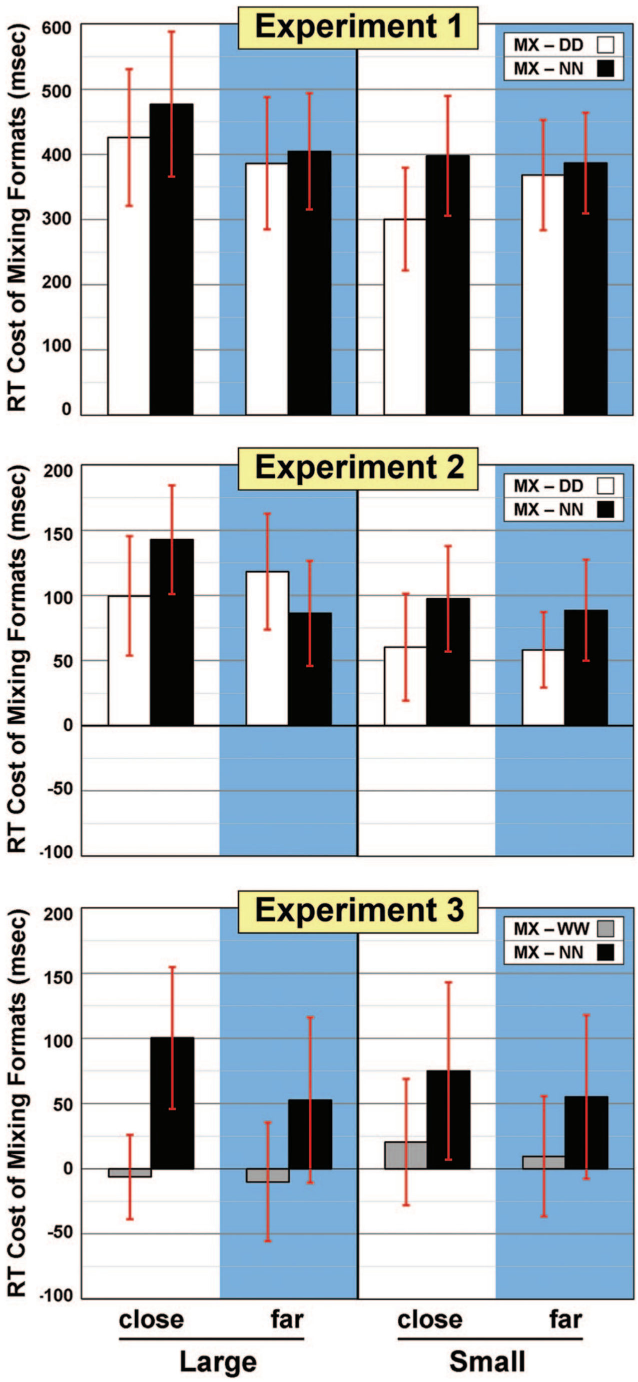

Figure 2. Comparison of the cost (mean difference, in milliseconds) of mixing formats. Error bars are 95% confidence intervals (all critical contrasts were within-subjects, two-tailed). Hence, if the lower bar crosses 0, there was no significant cost of mixing formats. MX = mixed-format (symbolic/nonsymbolic in Experiments 1–2, symbolic/symbolic in Experiment 3); DD = dot–dot; NN = numeral–numeral; WW = word–word; RT = response times.

0.59. There was no evidence of a speed–accuracy trade-off in terms of format-mixing costs. In particular, no significant negative slopes were observed when correlating the difference in errors between conditions (mixed-format vs. dot–dot) and the difference in response times (indeed, most correlations were weakly positive).

Experiment 2. It may have been that the difference between dot–dot and mixed-format performance in Experiment 1 arose not because of a weak link between the ANS and numerical symbols but due to the cost of switching between different perceptual input streams. Santens et al. (2010) showed that, unlike symbolic quantities, nonsymbolic quantities progress through an intermediate stage in the superior parietal lobe before arriving in the intraparietal sulcus. By contrast, previous work has demonstrated that occipitotemporal areas associated with visual word-form processing are engaged in early perceptual decoding of numerical symbols (Dehaene & Cohen 1995; Lyons & Ansari, 2009). It may simply be that the cost of mixing formats in Experiment 1 was driven by the inability to switch between these perceptual input streams—and hence may say nothing about numerical representation per se.

To ensure this was not the case, in Experiments 2–3, we chose an interstimulus interval (see Figure 1) that far exceeded (roughly doubled) the potential switch-cost window seen in Experiment 1 (maximum switch cost was 426 ms; see Figure 2, top, white bars). In addition, the 850 ms we allowed between Stimulus 1 onset and Stimulus 2 onset exceeded the maximum duration typically observed in visual attentional-blink paradigms (Kranczioch, Debener, Schwarzbach, Goebel, & Engel, 2005; Raymond, Shapiro, & Arnell, 1992). This further reduced the possibility that any evidence of mixing formats in Experiments 2–3 reflected switching between input processing streams.

In Experiment 2, response times were significantly longer for mixed-format than dot–dot trials in all categories: large–close: $t(20) = 4.53$, $p < .001$, $d = 0.99$; large–far: $t(20) = 5.56$, $p < .001$, $d = 1.21$; small–close: $t(20) = 3.07$, $p = .006$, $d = 0.67$; small–far: $t(20) = 4.18$, $p < .001$, $d = 0.91$; see Figure 2, middle, white bars). Error rates tended to be higher for mixed-format than dot–dot trials: large–close: $t(20) = 1.52$, $p = .144$, $d = 0.33$; large–far: $t(20) = 3.91$, $p = .001$, $d = 0.86$; small–close: $t(20) = 1.82$, $p = .084$, $d = 0.40$; small–far: $t(20) = 2.13$, $p = .046$, $d = 0.46$. As with Experiment 1, there was no evidence of a speed–accuracy trade-off in terms of format-mixing costs.

How might participants actually be translating between symbolic and nonsymbolic quantities? Izard and Dehaene (2008) demonstrated that participants show an underestimation effect when asked to quickly estimate the approximate number of dots in an array by stating aloud a symbolic, verbal label (e.g., "fifty"). In our data, there was evidence for systematic underestimation of dots, suggesting that participants may have been converting the dot arrays into verbal, symbolic labels to compare them directly with numerals. To demonstrate this, we looked separately at (mixed-format) trials for which the dot array was greater than the numeral and trials for which the opposite was true. When the dot array is greater, performance should be negatively affected by underestimation of the dot array because it should seem numerically closer to the numeral than it actually is. By contrast, when the dot array is numerically less, performance should be positively affected by underestimation because it should seem numerically further from the numeral than it actually is.

In Experiment 2 (no effects were significant for either measure in Experiment 1; $ps \geq .133$), participants tended to respond more slowly when the dot array was greater than the numeral (977 ms) than when the numeral was greater than the dot array (934 ms), $F(1, 20) = 11.75$, $p = .003$, $\eta^2 = .37$. Participants also tended to make more errors when the dot array was greater than the numeral (24.7%) than when the numeral was greater than the dot array (8.4%), $F(1, 20) = 53.67$, $p < .001$, $\eta^2 = .73$. Crucially, we observed longer response times for mixed-format versus dot–dot comparisons even for mixed-format cases in which the numeral was greater than the dot array: large–close: $t(20) = 3.07$, $p = .006$, $d = 0.67$; large–far: $t(20) = 2.20$, $p = .039$, $d = 0.48$; small–close: $t(20) = 3.05$, $p = .006$, $d = 0.67$; small–far: $t(20) = 2.57$, $p = .018$, $d = 0.56$. Underestimation of dot arrays cannot explain the observed mixing costs.

Experiment 3. In Experiment 3, we tested whether the cost of mixing formats observed in Experiments 1–2 might be due to mixing visual formats. Here, in contrast to the mixed-format conditions above, we predicted that mixing symbolic visual formats (numerals and number words) would not lead to worse performance than that seen for the worst performing single-format condition (word–word; see Table 1, bottom).

Performance did not significantly differ between mixed-format and word–word conditions either for response times, large–close: $t(20) = -0.41$, $p = .687$, $d = -0.09$; large–far: $t(20) = -0.46$, $p = .648$, $d = -0.10$; small–close: $t(20) = 0.88$, $p = .390$, $d = 0.19$; small–far: $t(20) = 0.43$, $p = .671$, $d = 0.09$ (see Figure 2, bottom, grey bars), or for error rates, large–close: $t(20) = -0.48$, $p = .636$, $d = -0.10$; large–far: $t(20) = -0.01$, $p = .992$, $d = 0.00$; small–close: $t(20) = 0.23$, $p = .818$, $d = 0.05$; small–far: $t(20) = 0.12$, $p = .907$, $d = 0.03$. Experiment 3 results are consistent with the hypothesis that switching between visual numerical formats—so long as both formats point to symbolic representations—does not incur the same cost as that arising when switching between symbolic and nonsymbolic numerical formats.

Comparing Experiments 2 and 3. As a final test, we asked whether the format-switching costs were significantly greater for translating between numerals and dots (Experiment 2) than for translating between numerals and number words (Experiment 3). In addition, we asked whether this cost was modulated by numerical size and/or numerical distance.

We submitted mixing costs for Experiment 2 (white bars, Figure 2, middle) and Experiment 3 (grey bars, Figure 2, bottom) to a 2 (Experiment: 2, 3; between-subjects) × 2 (Size: small, large; within-subjects) × 2 (Distance: close, far; within-subjects) ANOVA. For response times, the main effect of experiment was significant, $F(1, 40) = 13.61$, $p < .001$, $\eta^2 = .25$; the average format-mixing cost in Experiment 2 was 84 ms; the average cost in Experiment 3 was 3 ms. There was also a significant Experiment × Size interaction, $F(1, 40) = 11.60$, $p = .002$, $\eta^2 = .23$. For large quantities, format-mixing costs were 109 ms in Experiment 2 and −8 ms in Experiment 3, $F(1, 40) = 21.78$, $p < .001$, $\eta^2 = .35$. For small quantities, format-mixing costs were 59 ms in Experiment 2 and 7 ms in Experiment 3, $F(1, 40) = 5.25$, $p = .027$, $\eta^2 = .12$. There were no significant effects of distance (remaining Fs < 1). For error rates, the main effect of experiment was significant, $F(1, 40) = 18.96$, $p < .001$, $\eta^2 = .32$; the average format-mixing cost in Experiment 2 was 5.6% errors; the average cost in Experiment 3 was 0.0% errors (all other effects, $ps \geq .130$).

In sum, the cost of mixing symbolic and nonsymbolic quantities (numeral vs. dot array) is greater than the cost of mixing two symbolic quantities (numeral vs. number word). Furthermore, it appears that symbolic and nonsymbolic quantities are somewhat distinct even for small quantities, albeit more so for large quantities. The small quantities used here (1–4) are within the subitizing range for adults (Mandler & Shebo, 1982; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). Such quantities tend to be represented with high acuity as they do not exceed the capacity of visual short-term memory (Ansari, Lyons, van Eimeren, & Xu, 2007; Luck & Vogel, 1997; Pylyshyn, 2001). Hence, neither familiarity (even the large quantities were multiples of 10) nor representational acuity (i.e., sharpness of small-number tuning curves; Merten & Nieder, 2009; Piazza et al., 2004) fully explains the cost of switching between symbolic and nonsymbolic quantities.

Discussion

Experiments 1 and 2 provide clear evidence that numerical comparisons between symbolic and nonsymbolic quantities are considerably more difficult than comparisons of two nonsymbolic quantities. One might expect the comparison of a highly accurate stimulus (numeral) and an inaccurate stimulus (dot array) to be easier (or at least no worse) than the comparison of two inaccurate stimuli (two dot arrays). Our data suggest instead that a numeral does not provide direct access to an approximate sense of the quantity it represents. Rather, it appears that additional, inefficient processing is required to compare symbolic with nonsymbolic quantities.

Our results are partially consistent with the place-code model of symbolic number representation proposed by Verguts and Fias (2004), in which a numerical symbol is at least in part represented in terms of its relative ordinal position. Our results go further, however, by suggesting that numerical symbols operate primarily as an associative system in which relations between symbols come to overshadow those between symbols and their quantity referents and may even become devoid of a strong sense of nonsymbolic quantity per se (Deacon, 1997; Nieder, 2009). Thus, it will be important for future research to understand symbolic number representation in a way perhaps tied only indirectly to actual quantity referents (e.g., Lyons & Beilock, 2011). This may be especially interesting to consider in a developmental context and with respect to the individual differences that limit exactly how and when numerical symbols are best understood in conjunction with or separate from one's more intuitive number sense (Ansari, 2008; Holloway & Ansari, 2010; Lyons & Beilock, 2009; Santens et al., 2010). In one interesting example, Siegler and Opfer (2003) demonstrated that the mapping between numerical symbols and continuous visuospatial frames changes from logarithmic to linear and that this change is related to various improvements in other mathematical abilities (e.g., Booth & Siegler, 2008; Opfer & Siegler, 2007). Although the relation between a continuous visuospatial "mental number line" and both numerical symbols and the ANS remains a contentious one (Chen & Verguts, 2010; Gevers et al., 2010; Santens & Gevers, 2008; van Dijck, Gevers, Lafosse, Doricchi, & Fias, 2011), the Siegler and Opfer work points to one potential mechanism by which the symbolic number representation system itself changes. An intriguing possibility is that the transition from a logarithmic to linear mapping of symbolic numbers onto visual space is also a reflection of the process by which symbolic numbers become estranged from their ANS counterparts.

Another interesting example of the interaction between symbolic numbers and the ANS is the systematic underestimation of nonsymbolic quantities observed in Izard and Dehaene (2008; found also in the current work). This underestimation may be due in part to systematic inefficiencies in translating between representation systems, necessitated by the fact that symbolic and nonsymbolic representations of number have become estranged. The recalibration effect they observe may be seen as a corrective reweighting of this translation process. However, implementation of this reweighting appears to engender an additional processing cost; even for cases in which the dot array was numerically less than the numeral, we found a cost of mixing formats (Experiment 2 of the current work).

Here it is important to note it may be the case that number sense and numerical symbols were simply never associated with one another in the first place (Butterworth, 2010; Le Corre & Carey, 2007), and our data are broadly consistent with such a proposal. On the other hand, considerable neural evidence has accrued suggesting that the neural substrates underlying the ANS do overlap at least to some extent with those thought to underlie symbolic representations of number (Dehaene et al., 2003; Fias et al., 2003; Nieder & Dehane, 2009; Piazza et al., 2007; Santens et al., 2010). Furthermore, recent developmental evidence suggests that individual differences in ANS acuity are linked with symbol-based math abilities from a relatively young age (Gilmore et al., 2010; Halberda et al., 2008; McCrink & Spelke 2010; Piazza et al., 2010; Wagner & Johnson, 2011) and even into adulthood (Lyons & Beilock, 2011). If symbolic and nonsymbolic representations were never overlapping to begin with, the results from these studies could only be explained by an increase in the overlap between ANS and symbolic number representation over the course of development. Our data dispel this hypothesis: At least for adults, the mature endpoint involves two relatively distinct representation systems for symbolic and nonsymbolic numbers. Nevertheless, only future developmental work can unpack all the relevant mechanisms underlying numerical symbol learning at earlier ages. The specific question of how development shapes the relation between numerical symbols and the ANS is directly motivated by the current work.

On a broader note, a recent paper (Gebuis & Reynvoet, 2012; see also the supplemental materials to the current paper for additional methodological considerations raised by their work) suggests that nonsymbolic quantity is more grounded in low-level visual parameters than has been previously assumed. Because our data show that accessing an approximate sense of quantity from numerical symbols is much more difficult than previously thought, we argue that numerical symbols are more estranged from this perceptual, nonsymbolic grounding than has been previously assumed. In sum, we feel our results parallel those of Gebuis and Reynvoet, and the two papers together call into question previous assumptions about the nature of both symbolic and nonsymbolic representations of quantity.

In sum, our data speak to the interaction between an evolutionarily ancient (ANS) number system and a recent, culturally invented one. Although numerical symbols may or may not have co-opted ANS representations early in development, they appear to have emerged as a system distinct from the ANS—and perhaps a complex associative system unique unto itself. The data reported here plainly call into question the strength of the link in literate adults between numerical symbols and a sense of the quantities they are meant to represent. Future studies aimed at understanding the cognitive and neural basis of more complex math skills in particular should consider not only the commonalities across systems but also the unique properties that symbolic representations of number bring to the table.

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